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Non-integer viscoelastic constitutive law to model soft biological tissues to in-vivo indentation

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Purpose: During the last decades, derivatives and integrals of non-integer orders are being more commonly used for the description of constitutive behavior of various viscoelastic materials including soft biological tissues. Compared to integer order constitutive relations, non-integer order viscoelastic material models of soft biological tissues are capable of capturing a wider range of viscoelastic behavior obtained from experiments. Although integer order models may yield comparably accurate results, non-integer order material models have less number of parameters to be identified in addition to description of an intermediate material that can monotonically and continuously be adjusted in between an ideal elastic solid and an ideal viscous fluid.

Methods: In this work, starting with some preliminaries on non-integer (fractional) calculus, the "spring-pot", (intermediate mechanical element between a solid and a fluid), non-integer order three element (Zener) solid model, finally a user-defined large strain non-integer order viscoelastic constitutive model was constructed to be used in finite element simulations. Using the constitutive equation developed, by utilizing inverse finite element method and *in vivo* indentation experiments, soft tissue material identification was performed.

Results: The results indicate that material coefficients obtained from relaxation experiments, when optimized with creep experimental data could simulate relaxation, creep and cyclic loading and unloading experiments accurately.

Conclusions: Non-integer calculus viscoelastic constitutive models, having physical interpretation and modeling experimental data accurately is a good alternative to classical phenomenological viscoelastic constitutive equations.

Key words: *fractional calculus, indentation tests, inverse finite element analysis, soft tissue constitutive relation, viscoelasticity*

1. Introduction

The mathematics of "fractional" calculus dates back to the 17th century when Leibniz introduces the notation $D^n y = \frac{d^n y}{dx^n}$ for differentiation where n is a non-negative integer, and in 1695, L'Hospital in a letter to Leibniz asks what happens if the order of derivative, n, becomes ½? The response of Leibniz is interesting: "It is a paradox but some day useful consequences will be drawn" [29]. The non-integer order (commonly used word fractional is a misnomer) differential and integral calculus may take not only the fractional numbers, but it may include any arbitrary number as the order of differentiation or integration. As Leibniz predicted, this property of calculus becomes a powerful tool recently in various science and engineering applications. The theory of non-integer order calculus is thoroughly reviewed in many references, e.g., Ross [29], Oldham and Spanier [22], Podlubny [25], Bayın [1], Dalir and Bashour [7], Machado et al. [20].

The most popular expression of the non-integer order integration in notation of Davis [8] is the Riemann–Liouville non-integer order integral which is also called the "integro-differential" expression (equation 1).

$$ {}_aD_x^{-\beta} f(x) = \frac{1}{\Gamma(\beta)} \int_a^x (x-t)^{\beta-1} f(t)dt, \quad \mathrm{Re}(\beta) > 0 \,. \quad (1)$$

The subscripts a and x denote the limits of the integration, called the "terminals" and $\Gamma(\beta)$ is the

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